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Publication date:
2009

Citation for published version (APA):
Yang, L., & Shen, Q. (2009). *Extending Adaptive Interpolation: From Triangular to Trapezoidal*. 25-30.
http://hdl.handle.net/2160/3026

Extending Adaptive Interpolation: From Triangular to Trapezoidal

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Abstract

Fuzzy interpolative reasoning strengthens the power of fuzzy inference by enhancing the robustness of fuzzy systems and reducing systems complexity. However, during the interpolation process, it is possible that multiple object values for a common variable are inferred which may lead to inconsistency in interpolated results. A novel approach [10] was recently proposed for identification and correction of defective rules in the transformations computed for interpolation, thereby removing the inconsistencies. However, the implementation of this work is limited to rule models involving triangular fuzzy variables. This paper extends the adaptive approach as presented in [10], by introducing trapezoidal variables in the representation and manipulation of fuzzy rule models. This significantly improves the applicability of adaptive fuzzy interpolation reasoning, as many fuzzy systems are modelled with trapezoidal (as well as triangular) variables.

1 Introduction

Fuzzy rule interpolation significantly enhances the robustness of fuzzy reasoning. When given observations have no overlap with any antecedent values, no rule can be fired in classical inference. However, interpolative reasoning through a sparse rule base may still obtain certain conclusions and thus improve the applicability of fuzzy models. Also, with the help of fuzzy interpolation, the complexity of a rule base can be reduced by omitting those fuzzy rules which may be approximated from their neighboring rules. A number of important interpolating approaches have been presented in the literature, including [1, 2, 3, 6, 7, 8, 9]. In particular, the scale and move transformation-based approach can handle both interpolation and extrapolation which involve multiple fuzzy rules, with each rule consisting of multiple antecedents. This approach also guarantees the uniqueness as well as normality and convexity of the resulting interpolated fuzzy sets. However, it is possible that more than one object value of a single variable may be derived or observed in fuzzy interpolation. This implies that certain inconsistencies may result.

To address the aforementioned problem, recently, *adaptive interpolative reasoning* has been proposed [10]. This approach is capable of efficiently detecting inconsistencies, locating possible fault candidates and modifying the candidates in an effort to remove all the inconsistencies. It works by artificially viewing the interpolative inference procedures as system components, and then utilising assumption-based truth maintenance system (ATMS) [4] to record the dependencies between an interpolated value as well as contradictions and its proceeding interpolation components. From this, the classical algorithm of general diagnostic engine (GDE) [5] is employed to manipulate these sets of dependent components of contradictions to generate all possible defective rules.

However, the adaptive approach of [10] is limited in its implementation in that fuzzy models are assumed to involve only triangular fuzzy variables. Nevertheless, fundamentally, this is not restricted by the underlying approach. Having identified this fact, the work of [10] is herein extended to allow the use of fuzzy variables which are represented by trapezoidal membership functions. This will considerably widen the scope of the existing approach for adaptive fuzzy interpolation because in many practical applications of fuzzy systems, variables are typically not only represented in triangular membership functions but also in trapezoidal.

The rest of this paper is structured as follows. Section 2 reviews the mechanism of adaptive interpolative reasoning. Section 3 describes the extension of generalizing the previously proposed adaptive approach to covering interpolations involving trapezoidal membership functions. Section 4 gives an example to illustrate the utility of this work. Section 5 concludes the paper and points out important future research.

2 Adaptive Interpolative Reasoning

Adaptive interpolative reasoning [10] provides a way to ensure inference results being consistent during the fuzzy interpolative process. Given a certain fuzzy contradiction metric, a β_0-*contradiction* is a contradiction whose corresponding degree $\beta > \beta_0$ for a predefined threshold β_0 ($0 \leq \beta_0 \leq 1$). With the help of this concept, the adaptive approach can be summarized in Figure 1. Firstly, an interpolative reasoning tool performs inferences on a task and passes the inferred results over each step of interpolation to the ATMS for dependency-recording. Then, the ATMS relays any β_0-*contradictions* as well as their dependent fuzzy reasoning components to the GDE which diagnoses the problem and generates all possible component candidates. After that, a modification process takes place to correct a certain candidate to restore consistency. A brief description of each of these key methods is given below.

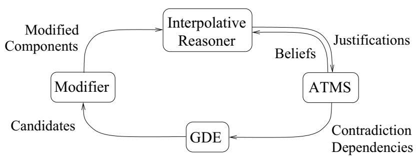

Figure 1: Adaptive interpolative reasoning

2.1 Truth maintenance

ATMS is utilized to record the dependency of the interpolated results as well as contradictions upon those fuzzy reasoning components from which they are inferred. Any ATMS node with an inferred proposition (i.e. a derived node in [5]) is represented by an ATMS justification as:

$$O, R_iR_j \Rightarrow C, \tag{1}$$

where R_iR_j stands for the fuzzy reasoning component containing the two neighboring rules R_i and R_j ($i \neq j$) that have been used to infer the outcome C from the observation O. Accordingly, a β_0-*contradiction* is represented as:

$$P, P' \Rightarrow_{\beta_0} \perp. \tag{2}$$

A label is a set of environments each supporting the associated node. An environment contains a minimal set of fuzzy reasoning components that jointly entail the concerned node, thereby describing how the node ultimately depends on those fuzzy reasoning components. An environment is said to be β_0-*inconsistent* if β_0-contradiction is derivable propositionally by the environment and a given justification. An environment is said to be $(1-\beta_0)$-*consistent* if it is not β_0-inconsistent.

The label of each node is guaranteed to be $(1-\beta_0)$-consistent, sound, minimal and complete by the label updating algorithm, except that the label of the special "false" node is guaranteed to be β_0-inconsistent rather than $(1-\beta_0)$-consistent. In particular, the label of the special "false" node gathers all β_0-inconsistent environments. Its corresponding label-updating process is given as follows. Whenever a β_0-contradiction is detected, each environment in its label is added into the label of "false" node and all such environments and their supersets are removed from the label of every other node. Also, any such environment which is a superset of another is removed from the label of the node "false".

2.2 Candidate generation

GDE [5] generates minimal candidates by manipulating the label of the specific "false" node. A candidate is a particular set of assumptions which may be responsible for the whole set of current contradictions. Because a β_0-inconsistent environment indicates that at least one of its assumption is faulty, a candidate must have a nonempty intersection with each β_0-inconsistent environment. Thus, each candidate is constructed by taking one assumption from each environment in the label of "false" node. Supersets removal then ensures such generated candidates to be minimal. In light of this, a successful correction of any single candidate will remove all the contradictions (see later).

2.3 Candidate modification

Consistency can be restored by successfully correcting any single candidate because each single candidate explains the entire set of current contradictions. Given a set of candidates, the modification procedure is shown in Figure 2. For convenience, in the rest of this paper, A^*_{ij} is used to denote the modified consequence of a culprit interpolated rule whose consequent value is A_{ij}, and $A^*_{ij}{}'$ and λ^*_{ij} are used to denote the corresponding modified intermediate rule consequence and the *relative placement factor* of A^*_{ij}, respectively. Suppose that the neighboring rules $A_{11} \Rightarrow A_{21}$ and $A_{1n} \Rightarrow A_{2n}$ are the two base rules used by a defective fuzzy reasoning component, that $A_{12}, A_{13}, ..., A_{1(n-1)}$ are observations or previously interpolated results located in between A_{11} and A_{1n}, and that A_{1j} ($2 \leq j \leq n-1$)

CONSISTENCYRESTORING($\mathbb{Q}$)
$\mathbb{Q}$, the candidate set in a FIFO descending queue in cardinality, each element of which is a set of fuzzy reasoning components (f);
MODIFY(f), the modification procedure for single fuzzy reasoning component (f). Return **true** when modification succeeds and return **false** otherwise.

```
success ← false
do
    C ← Dequeue(Q)
    foreach f ∈ C
        success ← MODIFY(f)
        if (success ==false)
            break
until ((success ==true) or (Q == ∅))
return success
```

Figure 2: The CONSISTENCYRESTORING procedure

is the middle most one. The modification procedure for single fuzzy reasoning component is summarized as follows.

1. Find out the rule ($A_{1j} \Rightarrow A_{2j}$) whose antecedent locates in the middle most of the neighborhood of the antecedents of the two base rules with respect to their representative values. Assume that the *relative placement factor* of its consequence λ_{2j} is modified to λ^*_{2j}.

2. Calculate the *correction rate pair* according to the *relative placement factor* modification of rule $A_{1j} \Rightarrow A_{2j}$:

$$\begin{cases} c^- = \frac{\lambda^*_{2j}}{\lambda_{2j}} \\ c^+ = \frac{1-\lambda^*_{2j}}{1-\lambda_{2j}}. \end{cases} \tag{3}$$

3. Calculate the modified *relative placement factors* of consequences of all other interpolated rules which are generated based on the same defective interpolative reasoning component as per the *correction rate pair* computed above, where $i \in \{2, 3, ..., j-1\}$ and $k \in \{j+1, j+2, ..., n-1\}$:

$$\begin{cases} \lambda^*_{2i} = \lambda_{2i} \cdot c^- \\ 1 - \lambda^*_{2k} = (1 - \lambda_{2k}) \cdot c^+. \end{cases} \tag{4}$$

4. Calculate the modified consequences of all interpolated rules which are generated based on the same defective interpolative reasoning component as per their modified *relative placement factors*:

$$\begin{cases} {A^*_{2x}}' = (1 - \lambda^*_{2x})A_{21} + \lambda^*_{2x}A_{22} \\ T({A_{1x}}', A_{1x}) = T({A^*_{2x}}', A^*_{2x}), \end{cases} \tag{5}$$

where $x \in \{2, 3, ..., n-1\}$, and $T(A', A)$ represents scale and move transformations from fuzzy set A' to A.

5. Restrict the modified consequence to be consistent with the context. Suppose that m object values $A_{i1}, A_{i2}, ..., A_{im}$ are obtained for variable x_i. If they are $(1 - \beta_0)$-consistent, they must satisfy:

$$\bigcap_{j=1}^{m} (A_{ij})_{\beta_0} \neq \varnothing, \tag{6}$$

where $(A_{ij})_{\beta_0}$ denotes the β_0-*cut* of A_{ij}.

6. Restrict the propagation of the modified consequence to be consistent with the context. For simplicity, let function $I(A_{ij}, R_l R_r) = A_{kj}$ denote the standard interpolation from the antecedent fuzzy set A_{ij} to the consequent value A_{kj}, based on the fuzzy reasoning component involving the neighboring rules R_l and R_r. Suppose that m object values $A_{i1}, A_{i2}, ..., A_{im}$ of variable x_i are modified which are located between the antecedent values of rules R_l and R_r, that the corresponding modified object values of variable x_k are A^*_{kj}, $j \in \{1, 2, ..., m\}$, and that n object values A_{kl}, $l \in \{1, 2, ..., n\}$, of variable x_k are already obtained one way or another. If the modified consequences A^*_{kj} are all $(1 - \beta_0)$-consistent, then they must satisfy:

$$\begin{cases} A^*_{kl} = I(A^*_{ij}, R_l R_r) \\ \left(\bigcap_{j=1}^{m} (A^*_{kj})_{\beta_0} \right) \cap \left(\bigcap_{l=1}^{n} (A_{kl})_{\beta_0} \right) \neq \varnothing. \end{cases} \tag{7}$$

7. Solve all these simultaneous equations generated above. The result is the modified solution which ensures inconsistency-free.

3 The Extension

It is potentially very useful to extend this adaptive approach to apply to fuzzy variables with trapezoidal fuzzy membership functions. This is because trapezoidal membership functions are also practically popular to model fuzzy systems apart from triangular. The extension is relatively straightforward due to the generality of ATMS, GDE, and the scale and move transformation-based interpolation, but there are still issues that require clarification, especially in the implementation of the approach. These points are discussed as follows.

3.1 Representative value and relative placement factor

The *representative value* captures the overall location of the fuzzy set. Consider a trapezoidal fuzzy set A_{ij}, denoted as $(p_{0(ij)}, p_{1(ij)}, p_{2(ij)}, p_{3(ij)})$, where $p_{0(ij)}$ and $p_{3(ij)}$ are the left and right coordinates of the start and end points of its support $(\forall x \in (p_{0(ij)}, p_{3(ij)}), \mu_{A_{ij}}(x) > 0)$ while $p_{1(ij)}$ and $p_{2(ij)}$ are the coordinates of the start and end points of its normal range $(\forall x \in (p_{1(ij)}, p_{2(ij)}), \mu_{A_{ij}}(x) = 1)$. The left support, right support and top support of A_{ij} are defined as $p_{1(ij)} - p_{0(ij)}$, $p_{3(ij)} - p_{2(ij)}$, and $p_{2(ij)} - p_{1(ij)}$ respectively. Note that this generic trapezoidal representation covers triangular fuzzy sets as its specific case, where $p_{1(ij)} = p_{2(ij)}$. Different definitions for representative values may be applied to meet different realistic requirements [6]. In order to be compatible with triangular representation situation, the *representative value* of a trapezoidal fuzzy set is defined as follows:

$$\mathrm{Rep}(A_{ij}) = \frac{1}{3}(p_{0(ij)} + \frac{p_{1(ij)} + p_{2(ij)}}{2} + p_{3(ij)}). \quad (8)$$

The *relative placement factor* reflects the relative location of the interpolated rule compared to its neighboring rules, which is calculated from the *representative values* of the relevant fuzzy sets. The *relative placement factor* λ_{ij} of the antecedent (or consequence) A_{ij} of an interpolated rule, with respect to its two neighboring rule antecedents (or consequences) A_{im} and A_{in}, is defined as the the ratio of $d(A_{im}, A_{ij})$ to $d(A_{im}, A_{in})$:

$$\lambda_{ij} = \frac{d(A_{im}, A_{ij})}{d(A_{im}, A_{in})} = \frac{d(\mathrm{Rep}(A_{im}), \mathrm{Rep}(A_{ij}))}{d(\mathrm{Rep}(A_{im}), \mathrm{Rep}(A_{in}))}, \quad (9)$$

where $d(A_{ix}, A_{iy})$ is the distance between fuzzy sets A_{ix} and A_{iy} (for a given distance metric).

3.2 Interpolation with trapezoidal fuzzy variables

Let x_i, $i \in \{1, 2, ..., n\}$, be a variable and $A_{i1}, A_{i2}, ..., A_{im_i}$ be the fuzzy sets in the domain of x_i. If $A_{11} \Rightarrow A_{21}$ and $A_{12} \Rightarrow A_{22}$ are two adjacent fuzzy rules in a sparse rule base, given an observed object value A_{13} of variable x_1, which does not match any existing rule and which is located between fuzzy sets A_{11} and A_{12}, the object value A_{23} of variable x_2 can be derived through fuzzy interpolative reasoning. The procedure of calculating A_{23} is summarized as follows:

1. Calculate the antecedent value of the intermediate rule ${A_{13}}' = ({p_{0(13)}}', {p_{1(13)}}', {p_{2(13)}}', {p_{3(13)}}')$ which has the same *representative value* as the observation A_{13}. For this, the *relative placement factor* λ_{13} of the observation A_{13} is calculated first according to (9) with respect to its flanks A_{11} and A_{12}. Then:

$$\begin{cases} {p_{0(13)}}' = (1 - \lambda_{13})p_{0(11)} + \lambda_{13}p_{0(12)} \\ {p_{1(13)}}' = (1 - \lambda_{13})p_{1(11)} + \lambda_{13}p_{1(12)} \\ {p_{2(13)}}' = (1 - \lambda_{13})p_{2(11)} + \lambda_{13}p_{2(12)} \\ {p_{3(13)}}' = (1 - \lambda_{13})p_{3(11)} + \lambda_{13}p_{3(12)}, \end{cases} \quad (10)$$

which are collectively abbreviated to:

$${A_{13}}' = (1 - \lambda_{13})A_{11} + \lambda_{13}A_{12}. \quad (11)$$

2. Calculate the consequence of the intermediate rule ${A_{23}}'$ by analogy to the calculation of ${A_{13}}'$ except letting the *relative placement factor* λ_{23} of the conclusion A_{23} be equal to λ_{13}:

$$\lambda_{23} = \lambda_{13}. \quad (12)$$

Then, $${A_{23}}' = (1 - \lambda_{23})A_{21} + \lambda_{23}A_{22}. \quad (13)$$

By the first two steps, the intermediate inference rule ${A_{13}}' \Rightarrow {A_{23}}'$ is constructed.

3. Calculate the similarity degree between ${A_{13}}'$ and A_{13} through two steps of transformation. Let ${A_{13}}'' = ({p_{0(13)}}'', {p_{1(13)}}'', {p_{2(13)}}'', {p_{3(13)}}'')$ denote the fuzzy set generated by scale transformation. This transformation transforms the current bottom support $({p_{0(13)}}', {p_{3(13)}}')$ into a new bottom support $({p_{0(13)}}'', {p_{3(13)}}'')$, and the top support $({p_{1(13)}}', {p_{2(13)}}')$ into a new top support $({p_{1(13)}}'', {p_{2(13)}}'')$ while keeping the *representative value* and the ratio of left-support $({p_{0(13)}}'', {p_{1(13)}}'')$ to right-support $({p_{2(13)}}'', {p_{3(13)}}'')$ of the transformed fuzzy set ${A_{13}}''$ the same as those of its original. This transformation is measured by *scale rates* s_b and s_t, and *scale ratio* $\mathbb{S}$ which are calculated by:

$$\begin{cases} s_b = \frac{{p_{3(13)}}'' - {p_{0(13)}}''}{{p_{3(13)}}' - {p_{0(13)}}'} = \frac{p_{3(13)} - p_{0(13)}}{{p_{3(13)}}' - {p_{0(13)}}'} \\ s_t = \frac{{p_{2(13)}}'' - {p_{1(13)}}''}{{p_{2(13)}}' - {p_{1(13)}}'} = \frac{p_{2(13)} - p_{1(13)}}{{p_{2(13)}}' - {p_{1(13)}}'}; \end{cases} \quad (14)$$

$$\mathbb{S} = \begin{cases} \dfrac{\frac{{p_{2(13)}}'' - {p_{1(13)}}''}{{p_{3(13)}}'' - {p_{0(13)}}''} - \frac{{p_{2(13)}}' - {p_{1(13)}}'}{{p_{3(13)}}' - {p_{0(13)}}'}}{1 - \frac{{p_{2(13)}}' - {p_{1(13)}}'}{{p_{3(13)}}' - {p_{0(13)}}'}} \in [0, 1], \\ (\text{if } \frac{{p_{2(13)}}'' - {p_{1(13)}}''}{{p_{2(13)}}' - {p_{1(13)}}'} \geq \frac{{p_{3(13)}}'' - {p_{0(13)}}''}{{p_{3(13)}}' - {p_{0(13)}}'} \geq 0) \\ \dfrac{\frac{{p_{2(13)}}'' - {p_{1(13)}}''}{{p_{3(13)}}'' - {p_{0(13)}}''} - \frac{{p_{2(13)}}' - {p_{1(13)}}'}{{p_{3(13)}}' - {p_{0(13)}}'}}{\frac{{p_{2(13)}}' - {p_{1(13)}}'}{{p_{3(13)}}' - {p_{0(13)}}'}} \in [-1, 0], \\ (\textit{otherwise}). \end{cases} \quad (15)$$

Move transformation shifts the current bottom support from $({p_{0(13)}}'', {p_{3(13)}}'')$ to $(p_{0(13)}, p_{3(13)})$, and top support from $({p_{1(13)}}'', {p_{2(13)}}'')$ to $(p_{1(13)}, p_{2(13)})$ while keeping the same *representative value*, that is, transforming fuzzy set ${A_{13}}''$ to fuzzy set A_{13}. The *move ratio* $\mathbb{M}$ measures this transformation which is calculated by:

$$\mathbb{M} = \begin{cases} \dfrac{p_{0(13)} - {p_{0(13)}}''}{\frac{{p_{1(13)}}'' - {p_{0(13)}}''}{3}}, & p_{0(13)} \geq {p_{0(13)}}'' \\ \dfrac{p_{0(13)} - {p_{0(13)}}''}{\frac{{p_{3(13)}}'' - {p_{2(13)}}''}{3}}, & \textit{otherwise.} \end{cases} \quad (16)$$

4. Transform ${A_{23}}'$ to A_{23} with the same transformation function T as used for transforming ${A_{13}}'$ to A_{13}: $T({A_{23}}', A_{23}) = T({A_{13}}', A_{13}). \quad (17)$

3.3 Candidate modification with trapezoidal fuzzy variables

The solution of defective fuzzy reasoning component modification results from solving a group of simultaneous equations which are linear for triangular representations. However, the complexity of this group of equations is raised to quadratic when using trapezoidal fuzzy sets. The reason for this is the introduction of scale ratio $\mathbb{S}$ (15). Following the description in last subsection, and taking a similar approach to the triangular representation situation, the scale rate s'_b between the intermediate consequence ${A_{23}}'$ and the transformed consequence ${A_{23}}''$ is set to that between its intermediate antecedent ${A_{13}}'$ and its transformed antecedent ${A_{13}}''$, but unlike triangular representation case, the scale rate s_t' between $A_{23}{}'$ and $A_{23}{}''$ is calculated under the condition that the scale ratio $\mathbb{S}$ between $A_{23}{}'$ and $A_{23}{}''$ is set to that between $A_{13}{}'$ and $A_{13}{}''$. This is followed by:

$$\begin{cases} s_b' = s_b, \\ s_t' = \dfrac{s_b'(s_t-s_b)(\frac{p_{3(23)}-p_{0(23)}}{p_{2(23)}-p_{1(23)}}-1)}{s_b(\frac{p_{3(13)}-p_{0(13)}}{p_{2(13)}-p_{1(13)}}-1)} + s_b', \quad (s_t \geq s_b) \\ s_t' = \frac{s_t}{s_b} s_b' \qquad (s_b \geq s_t). \end{cases} \tag{18}$$

It is clear that the above equation is quadratic. Although higher computational complexity is incurred, this extension is worthwhile due to the allowance of utilizing the practically popular trapezoidal fuzzy variables.

4 An Illustrative Example

To illustrate the potential of this extended adaptive interpolative reasoning method, the problem given in [10] is reconsidered such that the original triangular fuzzy variables are replaced with trapezoidal fuzzy sets. The rule base is given as follows:

R_1: $(x_1{=}A_{11}) \Rightarrow (x_2{=}A_{21})$; R_2: $(x_1{=}A_{12}) \Rightarrow (x_2{=}A_{22})$;
R_3: $(x_2{=}A_{23}) \Rightarrow (x_3{=}A_{31})$; R_4: $(x_2{=}A_{24}) \Rightarrow (x_3{=}A_{32})$;
R_5: $(x_2{=}A_{25}) \Rightarrow (x_4{=}A_{41})$; R_6: $(x_2{=}A_{26}) \Rightarrow (x_4{=}A_{42})$;
R_7: $(x_3{=}A_{33}) \Rightarrow (x_5{=}A_{51})$; R_8: $(x_3{=}A_{34}) \Rightarrow (x_5{=}A_{52})$;
R_9: $(x_4{=}A_{43}) \Rightarrow (x_5{=}A_{53})$; R_{10}: $(x_4{=}A_{44}) \Rightarrow (x_5{=}A_{54})$.

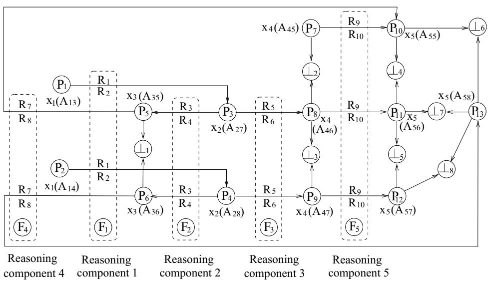

Figure 3: Discrepancy records in ATMS

Given $\beta_0 = 0.5$ and three observations, $x_1 = A_{13} = (7.0, 7.5, 8.0, 8.5)$, $x_1 = A_{14} = (7.6, 8.1, 8.6, 9.1)$ and $x_4 = A_{45} = (12.0, 12.5, 13.0, 13.5)$, the interpolation procedures are illustrated in Figure 3 and the original observations as well as interpolated results by scale and mover transformation-based interpolation technique are presented in Figure 4.

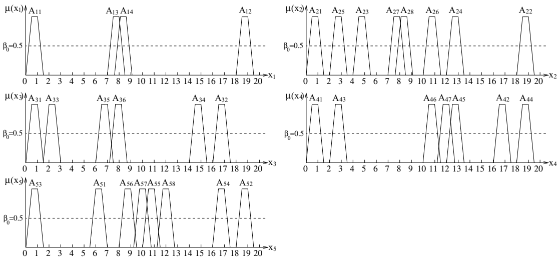

Figure 4: Fuzzy sets used in the example

4.1 Dependency recording by ATMS

In Figure 3, an arrow line flanked by two rules R_i and R_j represents a fuzzy reasoning component, which is denoted as R_iR_j, where R_i and R_j are the neighboring rules used for interpolation. ATMS nodes and contradictions are represented by circles. Particularly, each of F_i, $i \in \{1, 2, ..., 5\}$, is a node denoting a fuzzy reasoning component; each of P_j, $j \in \{1, 2, ..., 13\}$, is a node denoting a proposition; and each of $\perp_k$, $k \in \{1, 2, ..., 8\}$, denotes a β_0-contradiction. These ATMS nodes and contradictions are listed as follows, with all justifications omitted:

$F_1 : \langle R_1R_2, \{\{R_1R_2\}\}\rangle$; $F_2 : \langle R_3R_4, \{\{R_3R_4\}\}\rangle$;
$F_3 : \langle R_5R_6, \{\{R_5R_6\}\}\rangle$; $F_4 : \langle R_7R_8, \{\{R_7R_8\}\}\rangle$;
$F_5 : \langle R_9R_{10}, \{\{R_9R_{10}\}\}\rangle$; $P_1 : \langle x_1 = A_{13}, \{\{\}\}\rangle$;
$P_2 : \langle x_1 = A_{14}, \{\{\}\}\rangle$; $P_3 : \langle x_2 = A_{27}, \{\{R_1R_2\}\}\rangle$;
$P_4 : \langle x_2 = A_{28}, \{\{R_1R_2\}\}\rangle$;
$P_5 : \langle x_3 = A_{35}, \{\{R_1R_2, R_3R_4\}\}\rangle$;
$P_6 : \langle x_3 = A_{36}, \{\{R_1R_2, R_3R_4\}\}\rangle$; $P_7 : \langle x_4 = A_{45}, \{\{\}\}\rangle$;
$P_8 : \langle x_4 = A_{46}, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$P_9 : \langle x_4 = A_{47}, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$P_{10} : \langle x_5 = A_{55}, \{\{R_1R_2, R_3R_4, R_7R_8\}, \{R_9R_{10}\}\}\rangle$;
$P_{11} : \langle x_5 = A_{56}, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$P_{12} : \langle x_5 = A_{57}, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$P_{13} : \langle x_5 = A_{58}, \{\{R_1R_2, R_3R_4, R_7R_8\}\}\rangle$;
$\perp_1 : \langle \perp, \{\{R_1R_2, R_3R_4\}\}\rangle$; $\perp_2 : \langle \perp, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$\perp_3 : \langle \perp, \{\{R_1R_2, R_5R_6\}\}\rangle$;
$\perp_4 : \langle \perp, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$\perp_5 : \langle \perp, \{\{R_1R_2, R_5R_6, R_9R_{10}\}\}\rangle$;
$\perp_6 : \langle \perp, \{\{R_1R_2, R_3R_4, R_7R_8\}\}\rangle$;
$\perp_7 : \langle \perp, \{\{R_1R_2, R_3R_4, R_5R_6, R_7R_8, R_9R_{10}\}\}\rangle$;
$\perp_8 : \langle \perp, \{\{R_1R_2, R_3R_4, R_5R_6, R_7R_8, R_9R_{10}\}\}\rangle$.

In particular, a specific ATMS node "false", denoted by $P_\perp$, which collectively represents all the contradictions listed above from $\perp_1$ to $\perp_8$, is given as follows:

$P_\perp : \langle \perp, \{\{R_1R_2, R_3R_4\}, \{R_1R_2, R_5R_6\}\}\rangle$.

4.2 Candidate generation by GDE

Two minimal candidates can be generated according to the "false" node of the ATMS and its label $\{\{R_1R_2, R_3R_4\}, \{R_1R_2, R_5R_6\}\}$: $C_1 = [R_1R_2], C_2 = [R_3R_4, R_5R_6]$.

4.3 Candidate modification

In the running example, C_1 is chosen for modification first because candidate C_1 is smaller than C_2 in cardinality. Two rules have been interpolated based on this fuzzy reasoning component, both of which need to be modified:

IR_1: $(x_1{=}A_{13}){\Rightarrow}(x_2{=}A_{27})$; IR_2: $x_1{=}(A_{14}){\Rightarrow}(x_2{=}A_{28})$.

Following the modification procedure of single fuzzy reasoning component outlined in Section 2.3, the following simultaneous equation group can be set:

$c^- = \frac{\lambda_{28}^*}{\lambda_{28}}$; $c^+ = \frac{1-\lambda_{28}^*}{1-\lambda_{28}}$; $\lambda_{27}^* = \lambda_{27} \cdot c^-$;
$A_{27}^*{}' = (1-\lambda_{27}^*)A_{21} + \lambda_{27}^* A_{22}$;
$A_{28}^*{}' = (1-\lambda_{28}^*)A_{21} + \lambda_{28}^* A_{22}$;
$T(A_{13}{}', A_{13}) = T(A_{27}^*{}', A_{27}^*)$;
$T(A_{14}{}', A_{14}) = T(A_{28}^*{}', A_{28}^*)$; $(A_{27}^*)_{\beta_0} \cap (A_{28}^*)_{\beta_0} \neq \varnothing$;
$A_{35}^* = I(A_{27}^*, R_3R_4)$; $A_{36}^* = I(A_{28}^*, R_3R_4)$;
$(A_{35}^*)_{\beta_0} \cap (A_{36}^*)_{\beta_0} \neq \varnothing$; $A_{46}^* = I(A_{27}^*, R_5R_6)$;
$A_{47}^* = I(A_{28}^*, R_5R_6)$; $(A_{46}^*)_{\beta_0} \cap (A_{47}^*)_{\beta_0} \cap (A_{45})_{\beta_0} \neq \varnothing$;
$A_{55}^* = I(A_{35}^*, R_7R_8)$; $A_{56}^* = I(A_{46}^*, R_9R_{10})$;
$A_{57}^* = I(A_{47}^*, R_9R_{10})$; $A_{58}^* = I(A_{36}^*, R_7R_8)$;
$\cap_{j=5}^{8}(A_{5j}^*)_{\beta_0} \cap (A_{55})_{\beta_0} \neq \varnothing$.

Solving all these equations listed above simultaneously leads to no solution. Therefore, candidate C_1 is discarded and C_2 is then taken for tentative modification. Four rules have been interpolated through the two fuzzy reasoning components that comprises the candidate, which need to be modified:

IR_3: $(x_2{=}A_{27}){\Rightarrow}(x_3{=}A_{35})$; IR_4: $(x_2{=}A_{28}) \Rightarrow (x_3{=}A_{36})$;
IR_5: $(x_2{=}A_{27}){\Rightarrow}(x_4{=}A_{46})$; IR_6: $x_2{=}(A_{28}){\Rightarrow}(x_4{=}A_{47})$.

The following equations can be set according to the modification procedure of single fuzzy reasoning component outlined in Section 2.3.

$$c^-_{R_3R_4} = \frac{\lambda^*_{36}}{\lambda_{36}}; \quad c^+_{R_3R_4} = \frac{1-\lambda^*_{36}}{1-\lambda_{36}}; \quad \lambda^*_{35} = \lambda_{35} \cdot c^-_{R_3R_4};$$
$$A^*_{35}{}' = (1-\lambda^*_{35})A_{31} + \lambda^*_{35}A_{32};$$
$$A^*_{36}{}' = (1-\lambda^*_{36})A_{31} + \lambda^*_{36}A_{32};$$
$$T(A_{27}', A_{27}) = T(A^*_{35}{}', A^*_{35}); \quad T(A_{28}', A_{28}) = T(A^*_{36}{}', A^*_{36});$$
$$c^-_{R_5R_6} = \frac{\lambda^*_{46}}{\lambda_{46}}; \quad c^+_{R_5R_6} = \frac{1-\lambda^*_{46}}{1-\lambda_{46}};$$
$$(1-\lambda^*_{47}) = (1-\lambda_{47}) \cdot c^+_{R_5R_6};$$
$$A^*_{46}{}' = (1-\lambda^*_{46})A_{41} + \lambda^*_{46}A_{42};$$
$$A^*_{47}{}' = (1-\lambda^*_{47})A_{41} + \lambda^*_{47}A_{42};$$
$$T(A_{27}', A_{27}) = T(A^*_{46}{}', A^*_{46}); \quad T(A_{28}', A_{28}) = T(A^*_{47}{}', A^*_{47});$$
$$(A^*_{35})_{\beta_0} \cap (A^*_{36})_{\beta_0} \neq \varnothing; \quad (A^*_{46})_{\beta_0} \cap (A^*_{47})_{\beta_0} \cap (A_{45})_{\beta_0} \neq \varnothing;$$
$$A^*_{55} = I(A^*_{35}, R_7R_8); \quad A^*_{56} = I(A^*_{46}, R_9R_{10});$$
$$A^*_{57} = I(A^*_{47}, R_9R_{10}); \quad A^*_{58} = I(A^*_{36}, R_7R_8);$$
$$\cap^8_{j=5}(A^*_{5j})_{\beta_0} \cap (A_{55})_{\beta_0} \neq \varnothing.$$

Solving these simultaneous equations leads to one solution which is illustrated in Figure 5. It is clear from this figure that there is no β_0-contradiction any more and thus consistency has been restored. This means that the original inconsistent interpolation process has been corrected with consistent interpolated results throughout.

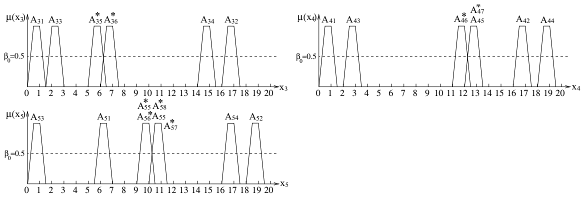

Figure 5: The solution for the running example

5 Conclusion

This paper has extended the recent work on adaptive interpolative reasoning, by allowing fuzzy variables to be represented by trapezoidal membership functions (instead of just in the triangular form). The approach uses the classical ATMS-based GDE approach to detect and isolate faults during the process of fuzzy interpolation. It modifies the identified culprit interpolated rules in an effort to restore consistency. The working of this method is illustrated with a practically significant example.

While the proposed approach is promising, further improvements may enhance its potential. Currently, all base rules which are provided in the initial rule base for interpolation are assumed to be totally true and are fixed. However, this may not be the case in certain real-world problems, despite the fact that it is a common assumption made in the literature of interpolative reasoning. Thus, it is important to extend the proposed work to allow base rules to become themselves diagnosable and modifiable. In addition, it is of great interest to develop an unified inconsistency diagnosis and fault correction mechanism for both standard fuzzy inference and fuzzy interpolation. Also, issues such as how to deal with rules with multiple antecedent variables and how to extend the proposed method to be used in fuzzy extrapolation require further research.

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